Pressure Vessel Engineering Ltd
120 Randall Drive, Suite B
Waterloo, Ontario, Canada
N2V 1C6

www.pveng.com
info@pveng.com
519-880-9808

Pressure Vessel Engineering Ltd. provides: ASME Vessel Code Calculations - Finite Element Analysis (FEA) - Solid Modeling / Drafting - Canadian Registration Number (CRN) Assistance

Finite Element Analysis Validation – SolidWorks Simulation 2010 x64 SP4.0

Pressure Vessel Engineering Ltd. validates each release of SolidWorks Simulation with a simple validation set to confirm that the program produces results comparable to Roark's Formulas results. Roark's formulas provide the exact derivation of the stresses of components of different geometry under several loading conditions. These formulas can be compared with FEA results.

Figure 1A provides the formulas for stresses in a heavy wall cylindrical disk or shell with ends caped and uniform internal pressure applied.

Figure 2A provides the formulas for stresses in a heavy wall spherical head with uniform internal pressure.

A test case is run and these (2) examples are calculated and compared in the following pages.

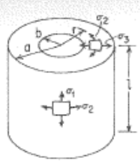
1. Cylindrical disk or shell 	1a. Uniform internal radial pressure q, longitudinal pressure zero or externally balanced; for a disk or a shell	$\sigma_1 = 0$ $\sigma_2 = \frac{qb^2(a^2+r^2)}{r^2(a^2-b^2)}, \quad (\sigma_2)_{max} = q\frac{a^2+b^2}{a^2-b^2}, \quad \text{at } r = b$ $\sigma_3 = \frac{-qb^2(a^2-r^2)}{r^2(a^2-b^2)}, \quad (\sigma_3)_{max} = -q, \quad \text{at } r = b$ $\tau_{max} = \frac{\sigma_2-\sigma_3}{2} = q\frac{a^2}{a^2-b^2}, \quad \text{at } r = b$ $\Delta a = \frac{q}{E}\frac{2ab^2}{a^2-b^2}, \quad \Delta b = \frac{qb}{E}\left(\frac{a^2+b^2}{a^2-b^2}+\nu\right), \quad \Delta l = \frac{-q\nu l}{E}\frac{2b^2}{a^2-b^2}$
	1b. Uniform internal pressure q, in all directions; ends capped; for a disk or a shell	$\sigma_1 = \frac{qb^2}{a^2-b^2}$ [σ_2, σ_3, $(\sigma_2)_{max}$, $(\sigma_3)_{max}$, and τ_{max} are the same as for case 1a] $\Delta a = \frac{qa}{E}\frac{b^2(2-\nu)}{a^2-b^2}$ $\Delta b = \frac{qb}{E}\frac{a^2(1+\nu)+b^2(1-2\nu)}{a^2-b^2}$ $\Delta l = \frac{ql}{E}\frac{b^2(1-2\nu)}{a^2-b^2}$

Fig 1A – Roark's formulas for stress in a heavy wall pipe

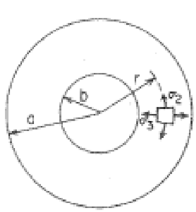
2. Spherical 	2a. Uniform internal pressure q	$\sigma_1 = \sigma_2 = \frac{qb^3}{2r^3}\frac{a^3+2r^3}{a^3-b^3}, \quad (\sigma_1)_{max} = (\sigma_2)_{max} = \frac{q}{2}\frac{a^3+2b^3}{a^3-b^3} \quad \text{at } r = b$ $\sigma_3 = \frac{-qb^3}{r^3}\frac{a^3-r^3}{a^3-b^3}, \quad (\sigma_3)_{max} = -q \quad \text{at } r = b$ $\tau_{max} = \frac{q3a^3}{4(a^3-b^3)} \quad \text{at } r = b$ The inner surface yields at $q = \frac{2\sigma_y}{3}\left(1-\frac{b^3}{a^3}\right)$ (Ref. 20) $\Delta a = \frac{qa}{E}\frac{3(1-\nu)b^3}{2(a^3-b^3)}, \quad \Delta b = \frac{qb}{E}\left[\frac{(1-\nu)(a^3+2b^3)}{2(a^3-b^3)}+\nu\right]$ (Ref. 3)
	2b. Uniform external pressure q	$\sigma_1 = \sigma_2 = \frac{-qa^3}{2r^3}\frac{b^3+2r^3}{a^3-b^3}, \quad (\sigma_1)_{max} = (\sigma_2)_{max} = \frac{-q3a^3}{2(a^3-b^3)} \quad \text{at } r = b$ $\sigma_3 = \frac{-qa^3}{r^3}\frac{r^3-b^3}{a^3-b^3}, \quad (\sigma_3)_{max} = -q \quad \text{at } r = a$ $\Delta a = \frac{-qa}{E}\left[\frac{(1-\nu)(b^3+2a^3)}{2(a^3-b^3)}-\nu\right], \quad \Delta b = \frac{-qb}{E}\frac{3(1-\nu)a^3}{2(a^3-b^3)}$ (Ref. 3)

Fig 2A – Roark's formulas for stress in a heavy wall sphere

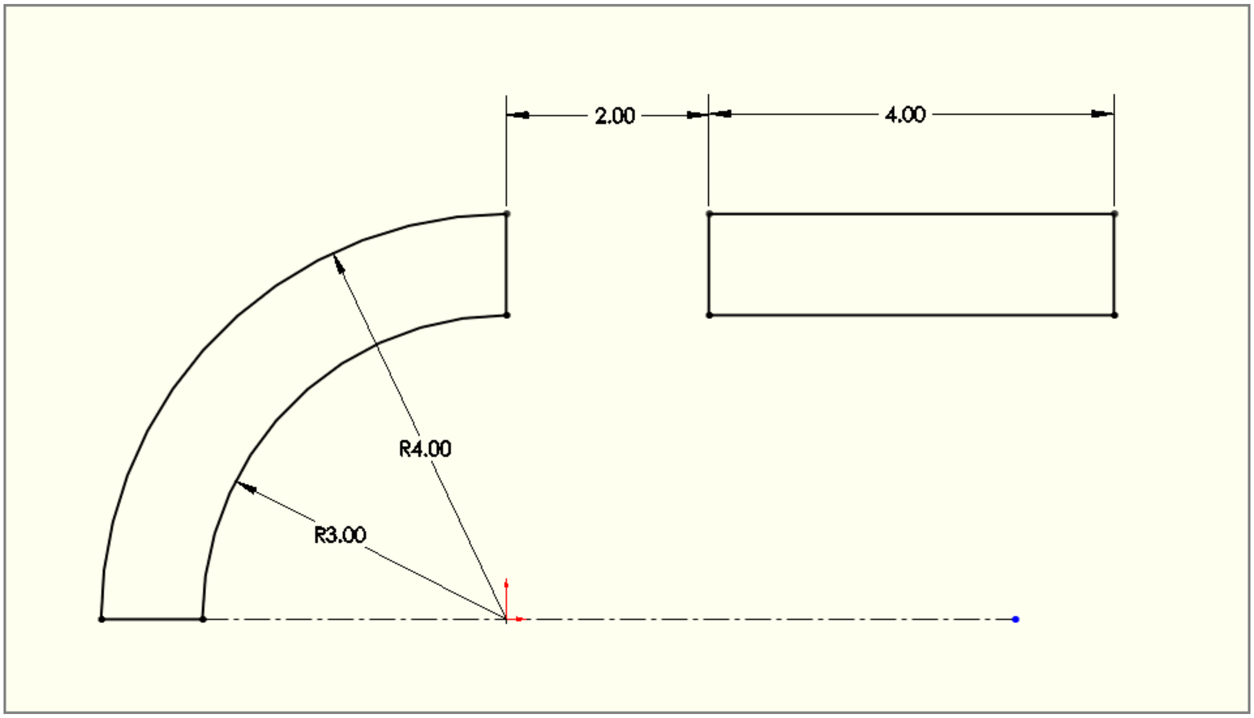


Fig 1B – Test Case Geometry

The geometry shown in Fig 1B is used for the heavy wall pipe & sphere comparison. The same geometry will be calculated by Roark's formulas and run in SW Simulation FEA.

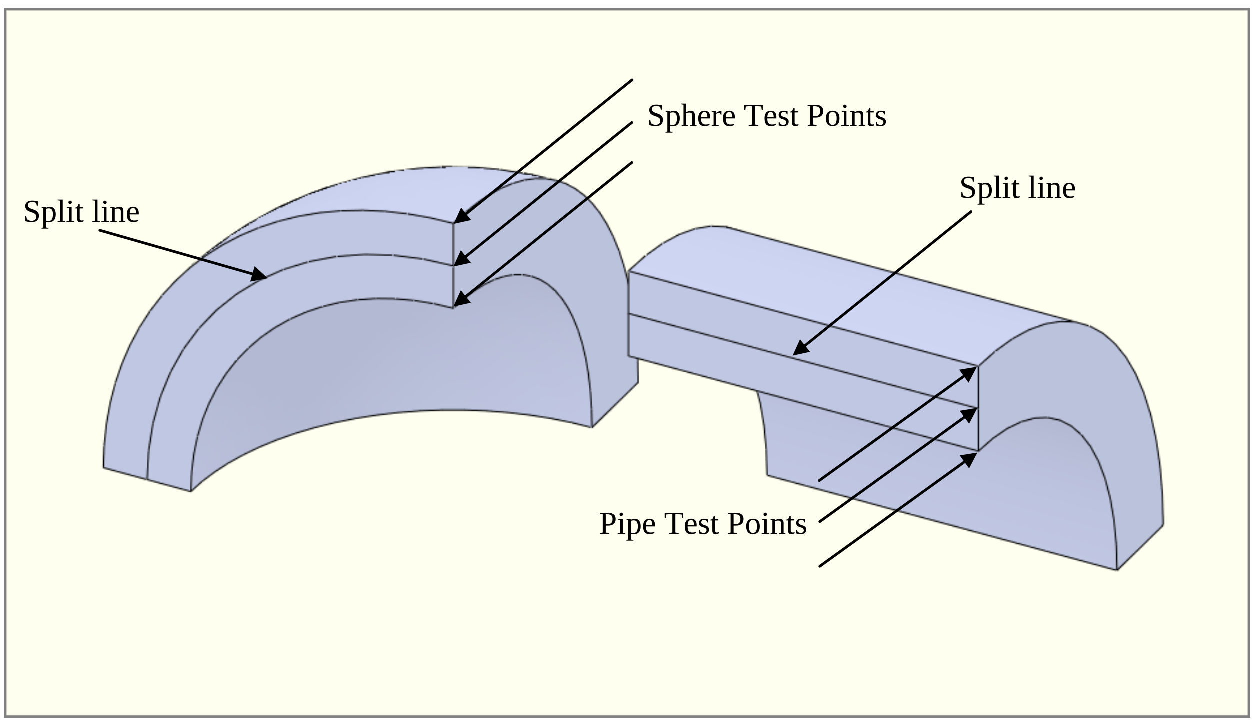


Fig 1C – Test Case Geometry

A split line is added to the model as shown in Fig 1C, this provides nodes at the midplane to probe so that stresses at the mid point may also be analyzed and compared to Roark's formulas results.

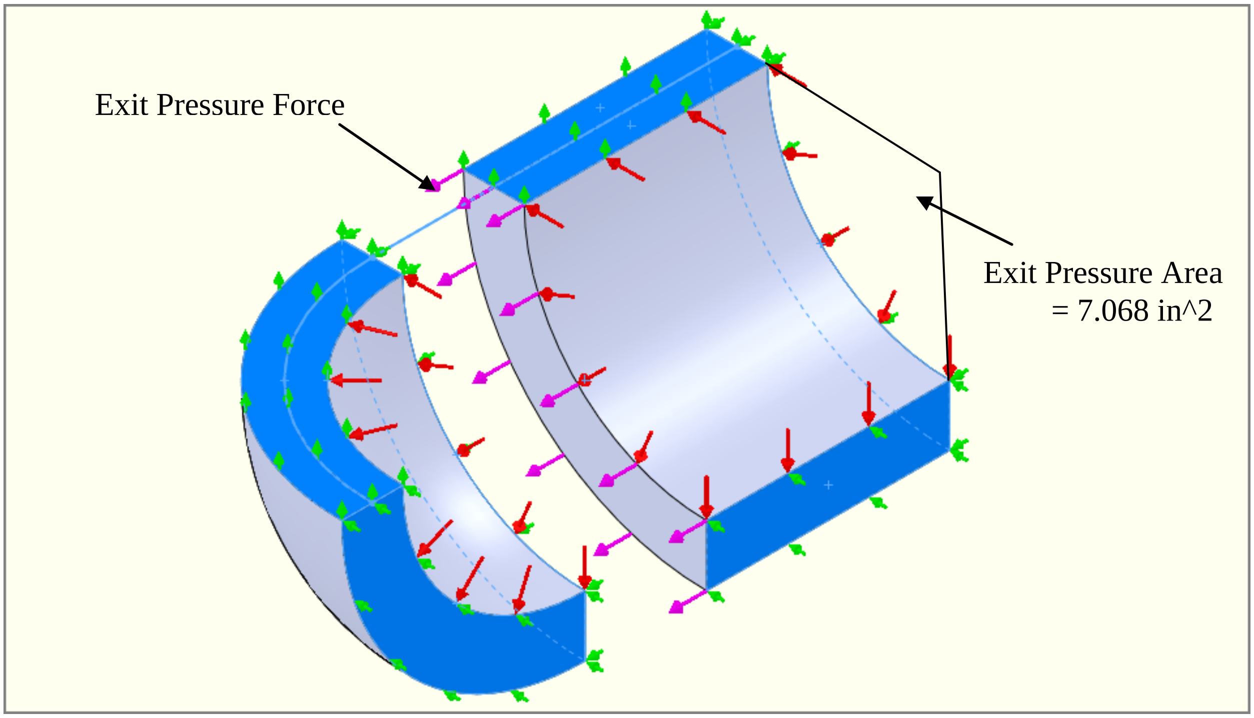


Fig 1D – Boundary Conditions & Loads Applied

Fig 1D shows symmetry boundary conditions applied to all faces, with the exception of (1) end of the pipe. An exit pressure force equal to the exit area of the pipe multiplied by the internal pressure is applied to this face (7.068in^2)*(1000psi) = 7068lb. 1000 psi is applied to all internal surfaces.

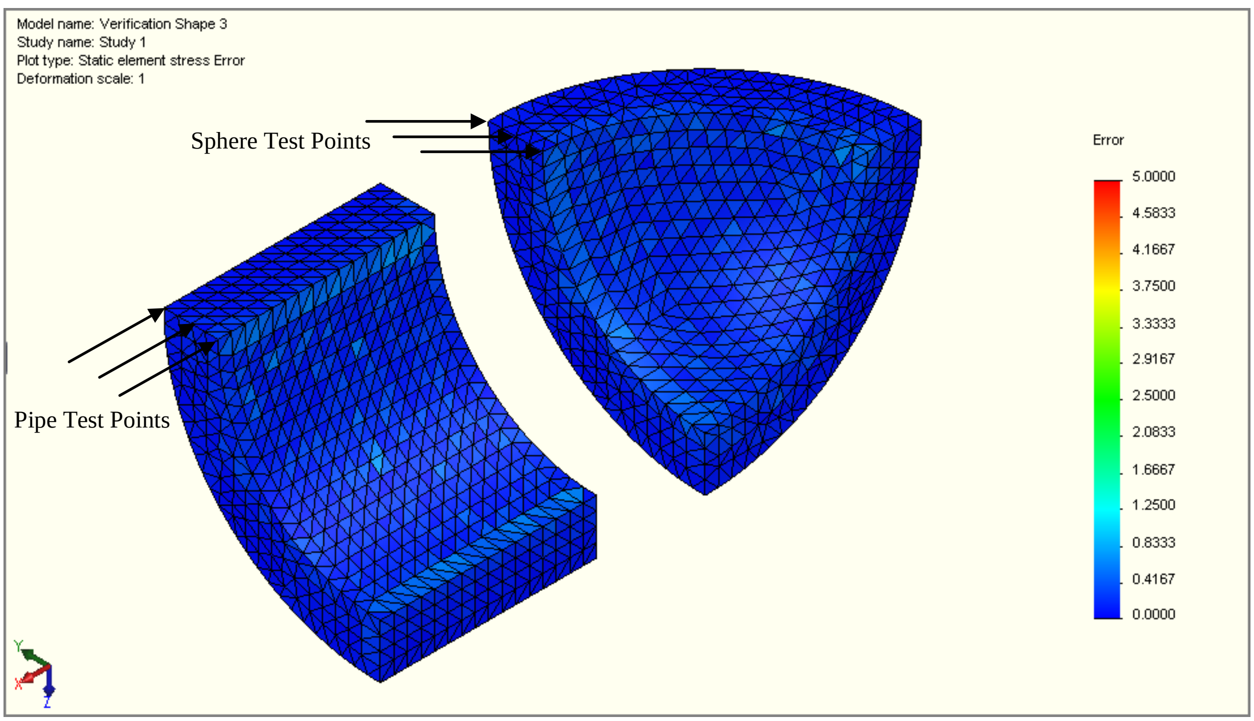


Fig 1E – Mesh & Error Plot

Fig 1E shows the resulting error plot for a ¼" mesh applied. Reported error is below 5% for the 2^{nd} order elements used. All nodes at test locations (in following pages) report error of less than 0.5%.

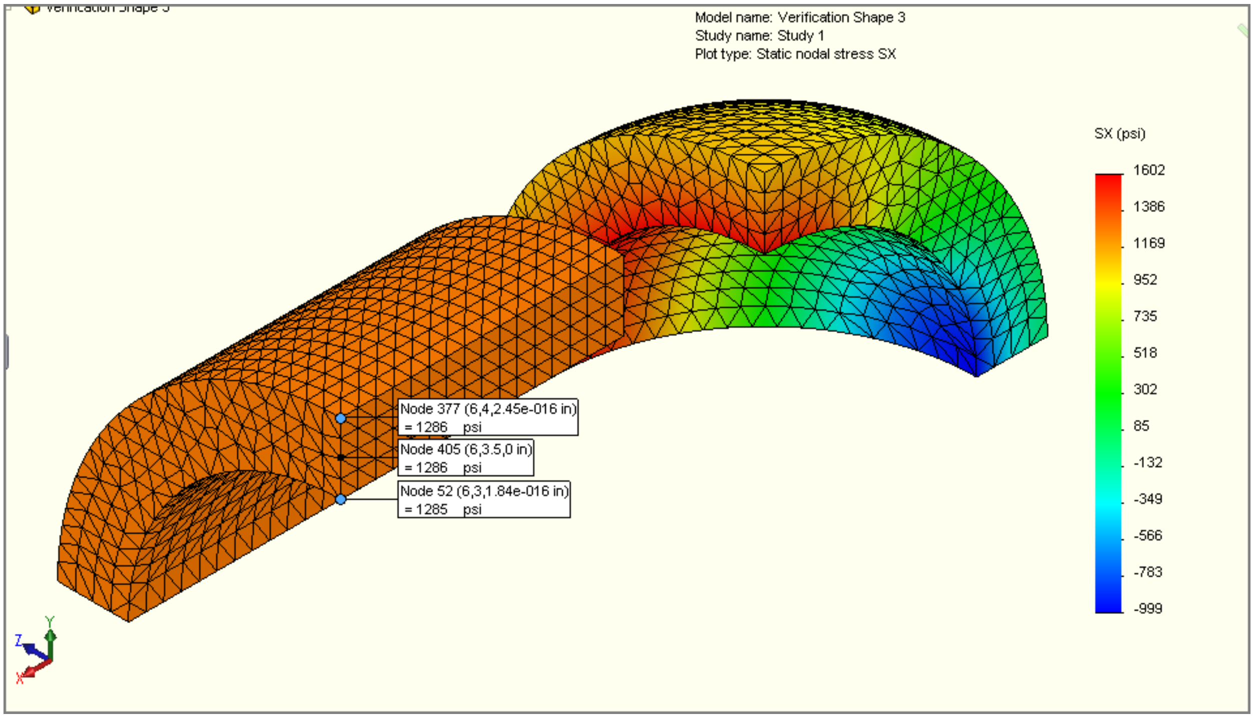


Fig 1F – Probe of Sx Stresses of pipe (Longitudinal)

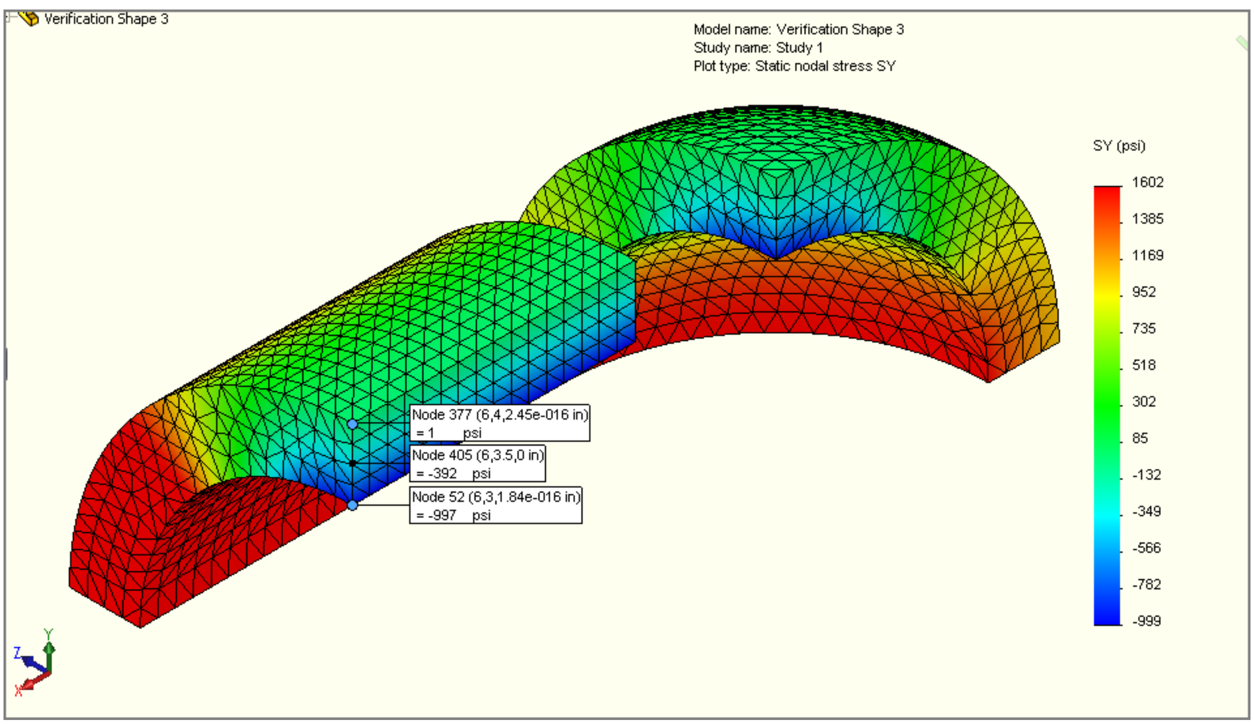


Fig 1G – Probe of Sy Stresses of pipe (Tangential)

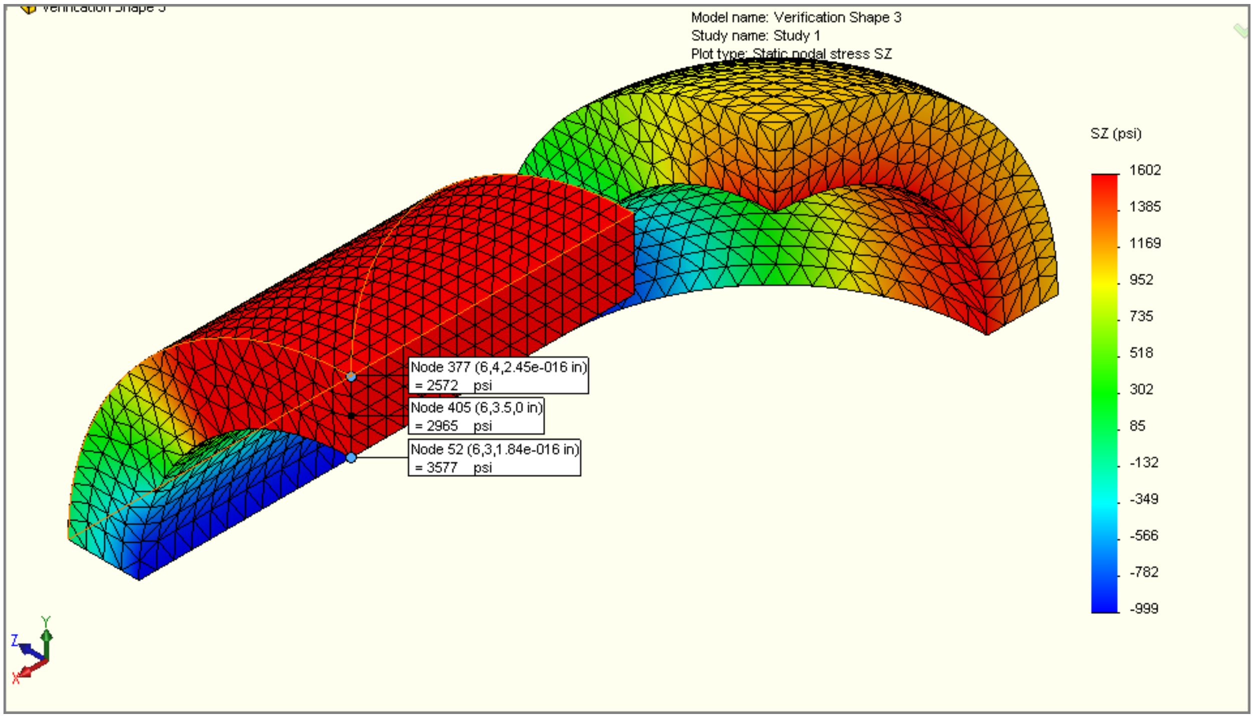


Fig 1H – Probe of Sz Stresses of pipe (Hoop)

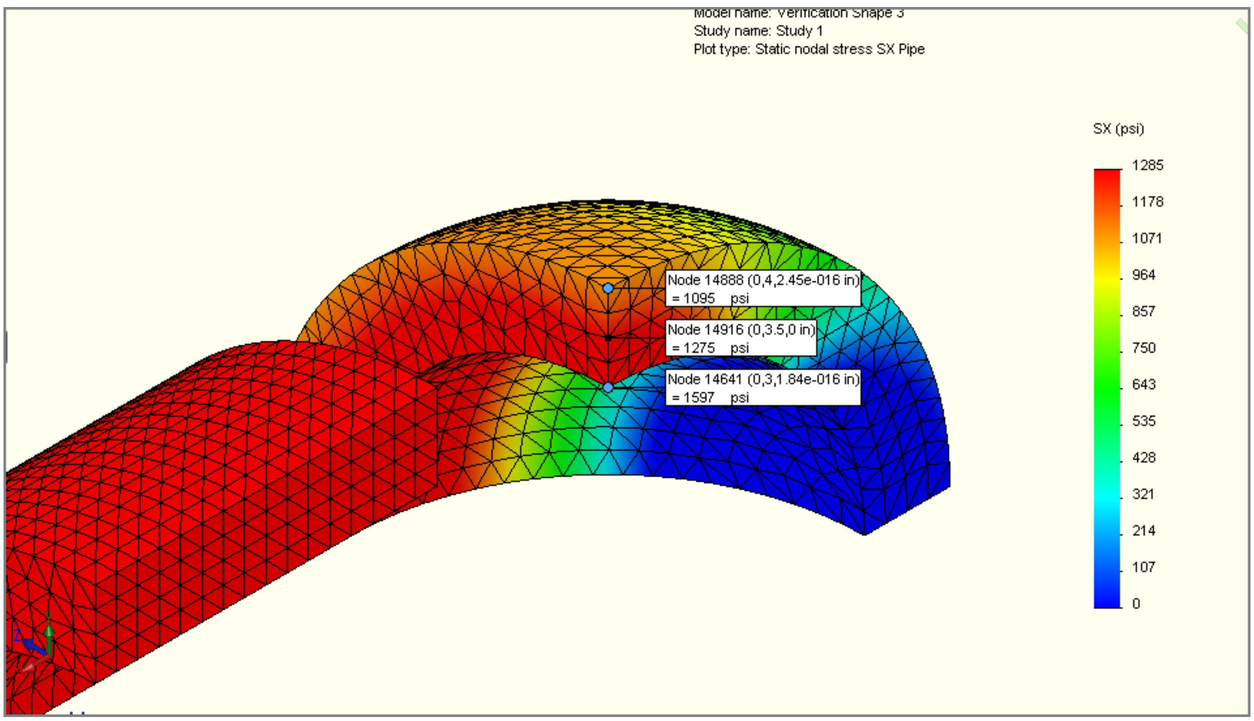


Fig 2B – Probe of Sx Stresses (Equal to Sz) of sphere

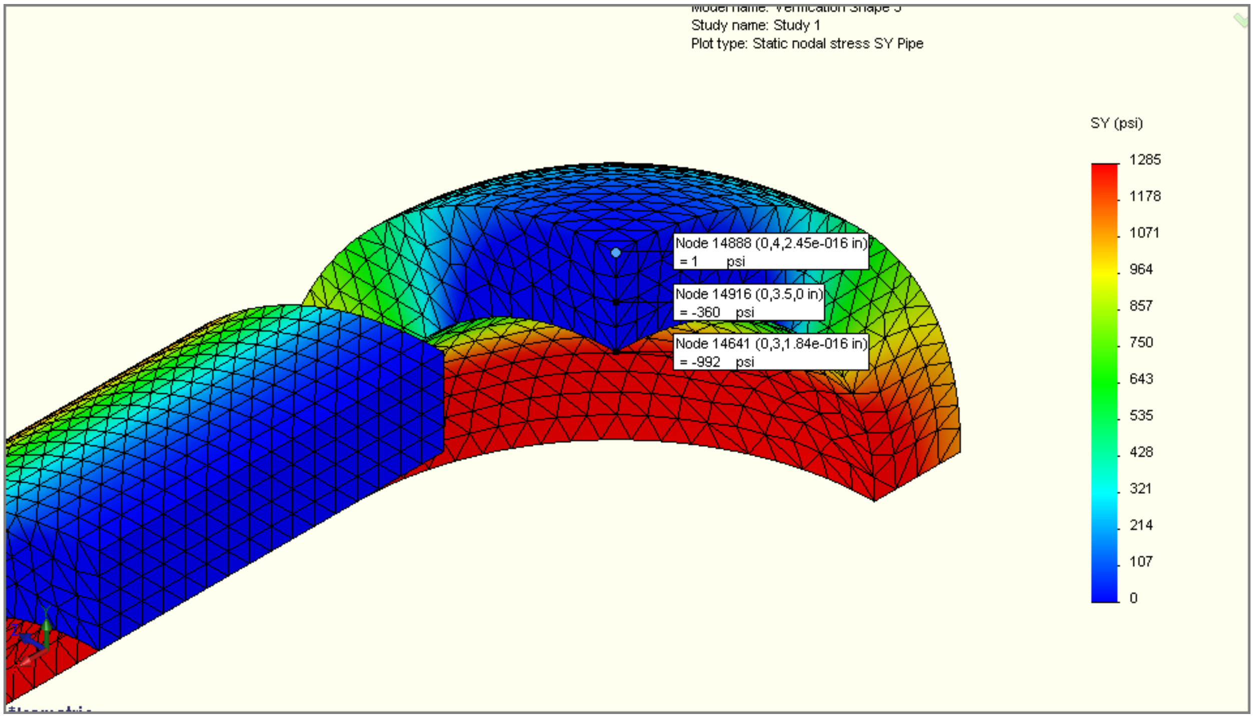


Fig 2C – Probe of Sx of sphere

The FEA results are compared to the results from Roark's formulas on the following pages. All results are extremely close to the theoretical values.

Conclusion: The FEA program provided results very close to the exact theoretical derivation provided by Roark's formulas. The maximum difference between Roark's and FEA is 0.8% for this test section.

Validation

For each version of SolidWorks Simulation released and used in Pressure Vessel Engineering Ltd's office, the 8" OD x 6" ID test specimen will be identically meshed and run. The difference reported between SW Simulation and Roark's will be expected to be less than 1.0% for the release to be accepted.

The release will be used once this validation is complete.

Job: PVE-3179
Test Shape: Verification Shape 3.sldprt
Excel File: COSMOSWorks Validation Calculations Rev.0.xls

Signoff:

2010 x64 SP4.0 ______ SW Simulation Version

Cameron Moore ______ Validated By

December 8, 2010 ______ Date